

Dynamic Load Restoration Considering the Interdependencies Between Power Distribution Systems and Urban Transportation Systems

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Abstract—After a major outage, mobile emergency resources (MERs) can be dispatched via the transportation system (TS) for service restoration to critical loads in the power distribution system (PDS). In this case study, the efficiency of service restoration in the PDS is associated with the traffic efficiency of the TS, and vice versa, because the PDS and TS are mutually coupled through traffic lights and MERs. This paper proposes a service restoration method considering interdependency between the PDS and TS, which is formulated as a mixed-integer linear program (MILP). The objective includes maximizing the efficiency of both PDS restoration and TS. By solving the MILP, the dynamic load restoration and MER dispatch strategies can be obtained. For the PDS, the availability of MERs, including mobile sources and repair crews, relates to their dispatch in the TS, and their relationship is formulated as mathematical models. For the TS, the dynamic traffic flow is modeled using the cell transmission model and the effect of traffic lights is considered. Case studies validate the effectiveness of the proposed method.

Index Terms—Mobile emergency resources, power distribution systems, resilience, service restoration, urban transportation systems.

NOMENCLATURE

Abbreviations

DG	Distributed generator.
MER	Mobile emergency resource.
MILP	Mixed-integer linear program.
PDS	Power distribution system.
TS	Transportation system.

Notation Associated with the TS

Sets and Parameters:

$\mathcal{C}$	Set of all cells, and $\mathcal{C}_O$, $\mathcal{C}_R$, $\mathcal{C}_I$ and $\mathcal{C}_S$ are sets of ordinary cells, source cells, intersection cells, and sink cells, respectively, with index a or b .
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$\mathcal{H}, \mathcal{T}$	Sets of cell connectors, and time intervals with the corresponding indices (a, b) and τ , respectively.
$\mathcal{P}$	Sets of different types of vehicles, and $\mathcal{P} = \{G, E, B, R, O\}$ indicating mobile generator and energy storage, electric bus, repair crew, and ordinary vehicles, respectively, with index c .
$\Omega(a)$, $\Omega^{-1}(a)$	Sets of successor cells and predecessor cells to cell a , respectively, $a \in \mathcal{C}$.
ΔT_{TS}	The time interval of the TS.
$Q_{a,\max}$	The maximum amount of flow that can flow into or out of cell a . ($Q_{a \in \mathcal{C}_R, \max}$ and $Q_{a \in \mathcal{C}_S, \max}$ are assumed as infinite or a large value).
$N_{a,\max}$	The maximum amount of flow that can be stored in cell a . ($N_{a \in \mathcal{C}_R, \max}$ and $N_{a \in \mathcal{C}_S, \max}$ are assumed as infinite or a large value).
δ_a	Congestion coefficient of cell a .
$X_{a,c}^0$	Initial flow in cell a for vehicles of type c at the initial time $\tau = 0$.
$Y_{(a,b),c}^0$	Initial flow in the connector (a, b) for vehicles of type c at the initial time $\tau = 0$.

Variables:

$x_{a,c}^\tau$	The traffic flow of type c vehicles in cell a at time τ .
$y_{(a,b),c}^\tau$	The traffic flow of type c vehicles moving from the cell a to the downstream cell b at time τ .
$\omega_{(a,a')}^\tau$	The states of the traffic signal in intersection (a, a') at time τ : if west-east direction is green and north-south direction is red, $\omega_{(a,a')}^\tau = 1$; otherwise, $\omega_{(a,a')}^\tau = 0$.

Notations Associated with the PDS

Sets and Parameters:

$\mathcal{L}$	Set of loads.
$\mathcal{N}_G, \mathcal{N}_E$, $\mathcal{N}_B$	Sets of connection buses for mobile generators, mobile storage systems, and electric buses, respectively.
$\mathcal{F}$	Set of all faulted zones, with the corresponding index l .
$\mathcal{G}_l$	Set of graphs including buses and lines of a faulted zone that cannot be energized due to faults, $\mathcal{G}_l = \langle \mathcal{N}_l, \mathcal{E}_l \rangle$.
$i \rightarrow j$	The directed line from i to j .

w_i	Weighted coefficient of load i , the larger of w_i , the more important of load i .
$P_{\min}, P_{\text{rate}}$	The lower and upper limitation of active output of each mobile generator.
$Q_{\min}, Q_{\text{rate}}$	The lower and upper limitation of reactive output of each mobile emergency generator.
$P_c^{\text{ch}}, P_c^{\text{dch}}$	The rated charge and discharge power of vehicles of type c , $c \in \{E, B\}$.
ΔT	Length of each time period in PDS.
E^0	The initial energy of each mobile generator.
$\text{SOC}_{c,\min}, \text{SOC}_{c,\max}$	The lower and upper State of Charge (SOC) of type c vehicles, $c \in \{E, B\}$.
SOC_c^0	The initiate SOC of type c vehicles, $c \in \{E, B\}$.
λ_i	The parameter of mobile energy storage system or electric bus connected to bus i that transforms the energy to SOC.
$\mathbf{Z}_{ij}$	Impedance matrix of the line (i, j) .
$\mathbf{v}_{i,\min}, \mathbf{v}_{i,\max}$	Vectors comprised of the square of minimal/maximal voltage magnitude at bus i in three phases.
$\mathbf{s}_{\text{load},i}$	Three-phase load demand of bus i .
M	A large positive real number.
N_c	The number of resource type c vehicles.
T_l	Time for repairing the faulted components in the faulted zone l .

Variables:

$p_{\text{gen},i}^t, q_{\text{gen},i}^t$	The active and reactive injection power of the mobile sources accession bus i at time t .
$s_{s,i}^t$	The output of the source of bus i at time t , $s_{s,i}^t = p_{s,i}^t + iq_{s,i}^t$.
$\mathbf{s}_i^t$	A three-dimensional vector variable indicating three-phase power injection of bus i at time t .
$\mathbf{v}_i^t, \mathbf{V}_i^t, \mathbf{i}_{ij}^t$	$\mathbf{v}_i^t$ is a complex three-dimensional vector variable indicating the three-phase voltage of bus i at time t , $\mathbf{V}_i^t := \mathbf{v}_i^t \mathbf{v}_i^{tH}$, $\mathbf{i}_{ij}^t$ is a complex three-dimensional vector variable indicating three-phase current of line (i, j) at time t .
$\mathbf{S}_{ij}^t$	$\mathbf{S}_{ij}^t := \mathbf{v}_i^t \mathbf{i}_{ij}^{tH}$.
$\mathbf{\Lambda}_{ij}^t$	A vector variable comprised of diagonal elements of $\mathbf{S}_{ij}^t$.
γ_i^t	Load status at time t : if load i is restored, $\gamma_i^t = 1$; otherwise, $\gamma_i^t = 0$.
β_l^t	Faulted zone status at time t : if faulted zone l is energized, $\beta_l^t = 1$; otherwise, $\beta_l^t = 0$.

I. INTRODUCTION

EXTREME events can damage power electrical infrastructures and cause large-scale long-lasting outages [1], [2]. Outages significantly impact critical infrastructures and lead to painful consequences such as social disorder. When the upper grids fail to support power distribution systems (PDSs), local resources, such as stand-alone distributed generators (DGs), energy storage systems, microgrids, and networked microgrids, together with mobile emergency recourses (MERs), can be utilized for service restoration to critical infrastructures for resilience enhancement [3]–[8]. MERs include mobile emergency sources and repair crews. They are dispatched through

the urban transportation system (TS) to destinations for service restoration in the PDS, and traffic lights depend on the power from the PDS to mitigate traffic congestions in the TS. MERs and traffic lights introduce interdependency between the PDS and TS [8], [9].

The dispatch of MERs, including routing and scheduling of mobile emergency sources and repair crews, have been studied by various scholars [7], [8], [10]–[15]. Mobile emergency sources provide power support to load restoration in PDSs and repair crews repair damaged components in PDSs to prompt the restoration process. In many papers, the modeling on TS is so simple that the impact of traffic conditions on travel time of MERs is ignored. In [10]–[13], the travel time is proportional to travel distance. In [14] and [15], the travel time is modeled as a random variable without considering the traffic conditions. However, in urban TS, the impact of traffic condition on MERs dispatch should not be ignored as traffic congestions may happen after extreme events. In [8], the dynamic traffic flow is modeled based on a cell transmission model thus the dynamic impact of traffic conditions on MERs dispatch is considered. Nevertheless, the impact of potential outages of traffic lights has not been considered.

Traffic lights play a crucial role in traffic management and affect the travel time of MERs in TS [9]. Outages of traffic lights can lead to traffic congestion, so the traffic lights should be restored by PDS as soon as possible. In [9], the impact of traffic lights after extreme events is considered in PDS planning to enhance the resilience of PDS coupled with urban TS. In [16], the traffic lights in expressways are considered in routing mobile de-icing devices for the PDS to fight ice storms. Limited work was reported on the dispatch of MERs considering traffic conditions managed by traffic lights.

After extreme events, the local resources and MERs can be coordinated for load restoration in the PDS. Generally, the decision-making of MER dispatches and load restoration are scheduled separately. The research on load restoration and MER dispatch integration is still in the early stages [7]. A synthetic mixed-integer linear program (MILP) model for PDS restoration and crew dispatch is proposed which achieves seamless coordination among switch operations, crew dispatch, and component repair in [7]. A two-stage PDS outage management method is proposed in [13] to co-optimize the repair crews, reconfiguration, and DG dispatch. To the best of our knowledge, the study on load restoration considering PDS and TS interdependency, including both MERs and traffic lights, has not been reported.

Service restoration, considering interdependency between the PDS and TS, is challenging. First, it involves multiple time scales. The traffic condition of the TS changes in seconds, while the restoration strategy of the PDS are implemented with an interval of minutes between restorative actions. Second, the two systems are mutually coupled through many traffic lights and MERs. The status of the two systems change dynamically and affects each other.

To tackle the above challenges, this paper proposes a dynamic load restoration method considering the interdependency between the PDS and TS. The major contributions include:

1) A multi-time-scale load restoration model is proposed and formulated as a MILP. The objective includes maximizing the weighted number of critical loads and minimizing the total travel time of vehicles.

2) For the PDS, the availability of MERs for load restoration is related to their dispatch in TS. That is, the arrival time of mobile sources and repair crews determines the availability of mobile sources and the faulted components for restoration.

3) For the TS, we model the dynamic traffic flow using the cell transmission model. The relationship between the outage of traffic lights and dispatch of MERs is modeled.

The remainder of this paper is organized as follows. Section II analyzes the interdependency between PDS and TS. The MILP model is formulated in Section III. Numerical results are illustrated in Section IV. Section V presents the conclusions.

II. INTERDEPENDENCY ANALYSIS

Assume that after an extreme event, such as a hurricane, the power infrastructures are damaged. The PDS is on outage and the utility power is unavailable. In this case, the resources are utilized for service restoration to critical loads in the PDS. The resources include local DGs and MERs, i.e., mobile emergency generators, mobile emergency storage systems, repair crews, and electric buses. MERs are dispatched via TS to destinations to support the PDS. The traffic lights in TS are also on outage. Out-of-service traffic lights can cause traffic jams. The interdependency between PDS and TS is shown in Fig. 1. The normal operation of traffic lights depends on load restoration in the PDS, which will affect the traffic condition of TS. In addition, the dispatch of MERs for the PDS depends on the traffic condition of the TS. The interdependency between the TS and PDS influences the load restoration process in the PDS.

An example of a PDS geographically corresponding to a TS is shown in Fig. 2, and the corresponding nodes between the PDS and TS are listed in the Appendix.

In the TS, mobile generators, energy storage systems, and repair crews are dispatched from the emergency center to the desired places, and electric buses are dispatched from the bus

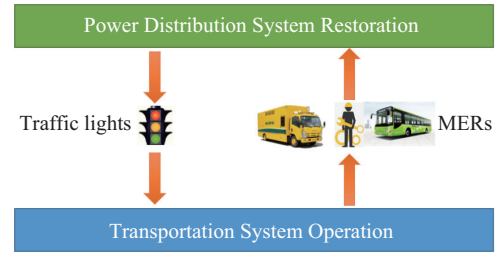


Fig. 1. The interdependency between the PDS and TS.

stations to the accession points. In this problem, the origin points for all MERs are given and the destinations would be determined in the dispatch and restoration strategies. For instance, in Fig. 2, mobile generators and energy storage systems can be accessed at nodes 6, 7, or 9 in the TS, corresponding to 8, 20, 27 in the PDS, and the specific destination for each mobile vehicle should be determined. The repair crews should be sent to node 10 in the TS, which corresponds to 29 in the PDS. In addition, the electric buses are sent from stations 13, 14, and 15 to the accession place 12, i.e., 26 in the PDS. The routing and scheduling of these MERs should be determined considering traffic conditions, which are affected by the normal function of traffic lights, while the traffic lights depend on service restoration by the PDS.

In the PDS, multiple faults are isolated. The local sources can be coordinated to restore loads to form electrical islands as large as possible [4]. Due to faults, some faulted zones are formed without sources or source accession points, such as the faulted zone in Fig. 2, which can only be restored after the crews repair the associated faults. The loads include critical loads and regular loads. The power demand of traffic lights is usually small. Assume that a set of traffic lights will be restored only if the connected electrical bus is energized and all loads connected to the bus are restored. For instance, the set of traffic lights in road junction 2 in the TS will be restored when the loads connected to bus 5 in the PDS are activated. This is because many distributed loads are connected to one bus through a transformer, while from the perspective of the

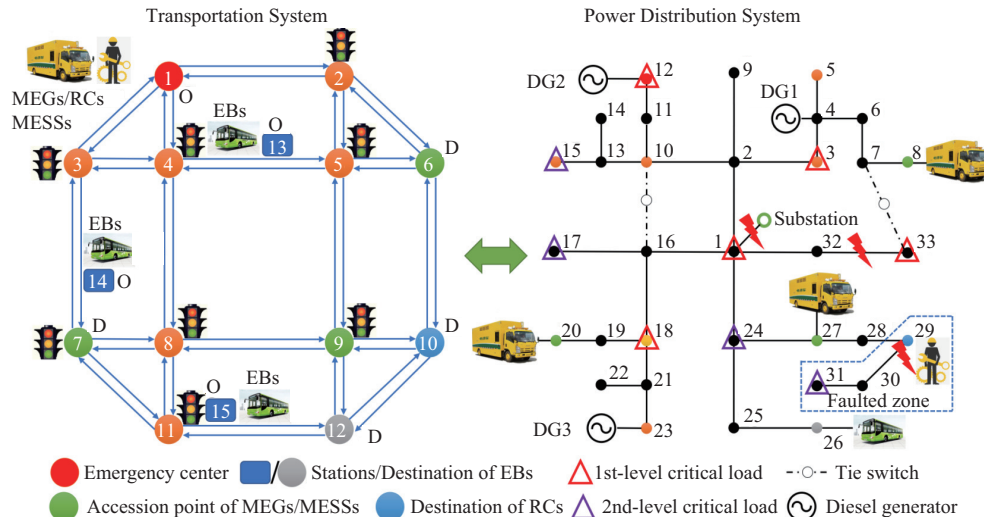


Fig. 2. An example of a TS corresponding to a PDS after an extreme event.

grid, only the switch status of the transformer is controllable. Therefore, we regard all loads connected to a bus as one load. For the PDS, the dynamic load restoration strategy should be obtained depending on the MERs dispatch in the TS. The load restoration strategy includes the restoration status of loads with respect to time, the outputs of power sources including mobile sources, and the line status.

The dynamic load restoration problem considering interdependency between the PDS and TS is formulated as a MILP, by solving the dispatch strategy of the MERs, the load restoration strategy can be obtained. The formulation of MILP will be introduced in the next section.

III. INTERDEPENDENCY MODELING

A. Time Division for the PDS and TS

As the whole process is with respect to time, the whole duration is, on average, divided into several intervals for the PDS and TS, as shown in Fig. 3.

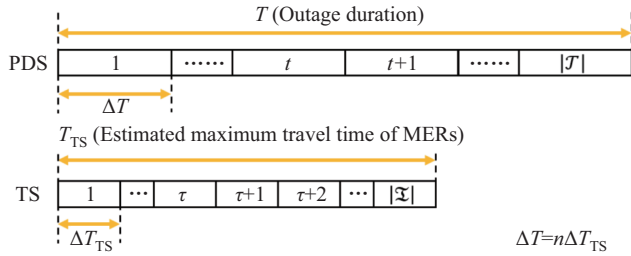


Fig. 3. Time division for the PDS and TS.

For PDS, the whole duration is the outage duration T , divided into $|T|$ periods indexed by t , while for TS, the whole duration is the estimated maximum travel time of MERs T_{TS} , divided into $|T|$ intervals indexed by τ . T_{TS} can be estimated according to historical data or experience of the decision-maker. Note that the granularity of time intervals for the PDS and TS is different. The interval ΔT for the PDS is in minutes and that for TS ΔT_{TS} is in seconds. This is because for PDS, the main task is to optimize the allocation of limited energy and frequent system state changing is unnecessary [17], while for TS, the traffic flow changes fast and shorter intervals for modeling the impact of traffic lights is essential [18]. In this paper, the interval for the PDS is n times longer than that for TS, where n is an integer.

B. Objective

The main objective is maximizing the cumulative service time for loads, representing the restoration efficiency. The secondary objective is minimizing the total travel time of all vehicles, indicating traffic efficiency in the TS, as shown in (1).

$$\max f = \sum_{t \in T} \sum_{i \in \mathcal{L}} w_i \gamma_i^t - w_0 \sum_{c \in \mathcal{P}} \sum_{\tau \in \mathcal{T}} \sum_{a \in \mathcal{C} \setminus \mathcal{C}_S} x_{a,c}^\tau \quad (1)$$

Note that the cumulative service time is $\sum_{t \in T} \sum_{i \in \mathcal{L}} w_i \gamma_i^t \Delta T$, where ΔT represents the time interval and can be omitted as it is a constant. Travel time has a nonnegative correlation with the density of cells, so the second term in

(1) represents the total travel time, where $a \in \mathcal{C} \setminus \mathcal{C}_S$ indicates all cells except sink cells. In (1), the value of weighting factor w_0 should be sufficiently small to ensure that the secondary objective does not interfere with the main objective.

C. Constraints in the TS

In this paper, the cell transmission model is used to simulate the spatial and temporal distribution of traffic flow and construct constraints in the TS [19], [20]. Four types of cells exist: source cell, ordinary cell, intersection cell, and sink cell, as shown in Fig. 4. The sets of the four types of cells are represented by $\mathcal{C}_O$, $\mathcal{C}_R$, $\mathcal{C}_I$, and $\mathcal{C}_S$. The source cell corresponds to the origin point of the vehicles, the ordinary cell to the road section or T-junctions, and the intersection cell to the intersections. The traffic lights are distributed in T-junctions and intersections. That is, the set of traffic light cells $\mathcal{C}_L = \mathcal{C}_I \cup \mathcal{C}_T$, where $\mathcal{C}_T \subseteq \mathcal{C}_R$ is the set of T-junction cells.

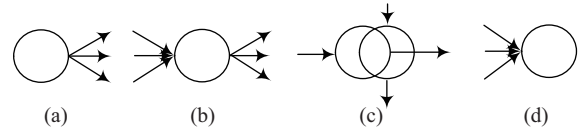


Fig. 4. Classification of cells. (a) Source cell. (b) Ordinary cell. (c) Intersection cell. (d) Sink cell.

1) Constraints on General Cells

As mentioned in Section II, 5 types of vehicles are considered. The origin points for MERs are set as source cells and the destination points for these vehicles are set as sink cells. Assume that when the extreme event happens, ordinary vehicles are distributed in the roads. The constraints for cells are formulated as follows:

$$x_{a,c}^\tau = x_{a,c}^{\tau-1} + \sum_{b \in \Omega^{-1}(a)} y_{(b,a),c}^{\tau-1} - \sum_{b \in \Omega(a)} y_{(a,b),c}^{\tau-1} \quad \forall a \in \mathcal{C}, \tau \in \mathcal{T} \setminus \{0\}, c \in \mathcal{P} \quad (2)$$

$$\sum_{b \in \Omega(a)} y_{(a,b),c}^\tau \leq x_{a,c}^\tau, \forall a \in \mathcal{C}, \tau \in \mathcal{T}, c \in \mathcal{P} \quad (3)$$

$$\sum_{c \in \mathcal{P}} \sum_{b \in \Omega(a)} y_{(a,b),c}^\tau \leq Q_{a,\max}, \forall a \in \mathcal{C}, \tau \in \mathcal{T} \quad (4)$$

$$\sum_{c \in \mathcal{P}} \sum_{b \in \Omega^{-1}(a)} y_{(b,a),c}^\tau \leq Q_{a,\max}, \forall a \in \mathcal{C}, \tau \in \mathcal{T} \quad (5)$$

$$\sum_{c \in \mathcal{P}} \sum_{b \in \Omega^{-1}(a)} y_{(b,a),c}^\tau \leq \delta_a \left(N_{a,\max} - \sum_{c \in \mathcal{P}} x_{a,c}^\tau \right), \quad \forall a \in \mathcal{C}, \tau \in \mathcal{T} \quad (6)$$

$$x_{a,c}^0 = X_{a,c}^0, y_{(a,b),c}^0 = Y_{(a,b),c}^0, \quad \forall a \in \mathcal{C}, (a,b) \in \mathcal{H}, c \in \mathcal{P} \quad (7)$$

$$\sum_{a \in \mathcal{C}_O} x_{a,c}^0 = \sum_{a \in \mathcal{C}_S} x_{a,c}^{|\mathcal{T}|}, \quad \forall c \in \mathcal{P} \quad (8)$$

Constraint (2) represents the traffic flow balance of each cell. Constraint (3) indicates that the traffic flow of vehicle type c from cell a to other cells is not larger than that in cell. Constraint (4) indicates the traffic flow of vehicle type c from

cell a to other cells is not larger than the maximum traffic flow of a . Constraint (5) indicates that the total traffic flow into cell a is not larger than the maximum traffic flow of a . Constraint (III-C1) shows that the total inflows of cell a do not exceed the available residual occupancy of a . This constraint deals with the situation when the traffic is congested. Constraints (III-C1) and (8) are initial and termination traffic flow states of each cell, respectively. In this paper, we consider the MERs are in source cells at initial state and sink cells at termination state.

2) Specific Constraints on Traffic Lights

In this paper, we consider the traffic lights distributed at intersections and T-junctions. The traffic outflow of traffic light cells highly depends on the state of the traffic lights. In order to illustrate this effect, a schematic diagram of intersection cells with traffic lights is shown in Fig. 5.

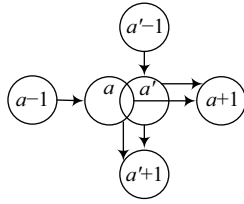


Fig. 5. A schematic diagram of intersections with traffic lights.

In Fig. 5, there are traffic lights at intersection (a, a') . Define the direction from west to east as a and the direction from north to south as a' [9]. The traffic flow constraints considering the operation of traffic lights are formulated as:

$$\sum_{c \in \mathcal{P}} \sum_{b \in \Omega(a)} y_{(a,b),c}^{\tau} \leq \gamma_{(a,a')}^{\tau} \omega_{(a,a')}^{\tau} Q_{a,\max} \quad (9)$$

$$a \in \mathcal{C}_I, \tau \in \mathfrak{T}$$

$$\sum_{c \in \mathcal{P}} \sum_{b \in \Omega(a')} y_{(a',b),c}^{\tau} \leq \gamma_{(a,a')}^{\tau} (1 - \omega_{(a,a')}^{\tau}) Q_{a',\max} \quad (10)$$

$$a' \in \mathcal{C}_I, \tau \in \mathfrak{T}$$

$$\sum_{c \in \mathcal{P}} \sum_{b \in \Omega(a)} y_{(a,b),c}^{\tau} \leq \delta \left(N_{a'+1,\max} - \sum_{c \in \mathcal{P}} x_{a'+1,c}^{\tau} \right) \quad (11)$$

$$a \in \mathcal{C}_I, \tau \in \mathfrak{T}$$

$$\sum_{c \in \mathcal{P}} \sum_{b \in \Omega(a')} y_{(a',b),c}^{\tau} \leq \delta \left(N_{a+1,\max} - \sum_{c \in \mathcal{P}} x_{a+1,c}^{\tau} \right) \quad (12)$$

$$a' \in \mathcal{C}_I, \tau \in \mathfrak{T}$$

where $\omega_{(a,a')}^{\tau}$ is a binary representing the state of the traffic signal. When $\omega_{(a,a')}^{\tau} = 1$, it is green for west-east direction and red for north-south direction, i.e., the traffic flow can go through the intersection from west to east (from cell $a - 1$ across cell a to cell $a + 1$) and no traffic flow from north to south (from cell $a' - 1$ across cell a' to cell $a' + 1$). $\gamma_{(a,a')}^{\tau}$ represents the load status of traffic lights (a, a') . Constraints (9) and (10) indicate that if traffic lights (a, a') are restored ($\gamma_{(a,a')}^{\tau} = 1$), the traffic flow of one direction is not larger than the maximum traffic flow and there is no traffic flow in another direction; otherwise, a congestion

may happen due to out-of-service traffic lights. Constraints (11) and (12) indicate that vehicles entering an intersection are reduced once the intersection is blocked by a static queue from another direction [18]. Take (11) as an example, in the case where $\sum_{c \in \mathcal{P}} x_{a'+1,c}^{\tau} = N_{a'+1,\max}$, indicating that the queue in direction a' blocks the intersection, then $\sum_{c \in \mathcal{P}} \sum_{b \in \Omega(a)} y_{(a,b),c}^{\tau} = 0$, i.e., vehicles from direction a cannot enter the intersection.

For T-junctions, similar with (9), the traffic flow constraints can be formulated as:

$$\sum_{c \in \mathcal{P}} \sum_{b \in \Omega(a)} y_{(a,b),c}^{\tau} \leq \gamma_a^{\tau} \omega_a^{\tau} Q_{a,\max}, \quad a \in \mathcal{C}_T, \tau \in \mathfrak{T} \quad (13)$$

D. Constraints in the PDS

In the PDS, the special constraints are those on MERs. When the mobile emergency sources arrive at the destination, the corresponding bus can inject power to the PDS for service restoration. In addition, after the repair crews reach the faulted site, the corresponding component and the associated faulted zone can be restored after repairing, with the topology changed. In the following, the constraints on mobile source accessions buses and faulted zones associated with repair crews are discussed in detail.

1) Constraints on Accession Buses of Mobile Sources

The power and energy capability of the mobile source accession buses depend on the number of arrived MERs and their operational features. In this paper, assume all the mobile generators are the same, and so are the mobile energy storage systems. The detailed constraints are as follows:

$$\begin{cases} x_{i,c}^t P_{\min} \leq p_{\text{gen},i}^t \leq x_{i,c}^t P_{\text{rate}} \\ x_{i,c}^t Q_{\min} \leq q_{\text{gen},i}^t \leq x_{i,c}^t Q_{\text{rate}} \end{cases} \quad (14)$$

$$c \in \{G\}, \forall i \in \mathcal{N}_G, t \in \mathcal{T}$$

$$-x_{i,c}^t P_c^{\text{ch}} \leq p_{\text{gen},i}^t \leq x_{i,c}^t P_c^{\text{dch}} \quad (15)$$

$$\forall c \in \{E, B\}, i \in \mathcal{N}_E \cup \mathcal{N}_B, t \in \mathcal{T}$$

$$\sum_{t \in \mathcal{T}} (\Delta T p_{\text{gen},i}^t) \leq x_{i,c}^{|T|} E^0, c \in \{G\}, \forall i \in \mathcal{N}_G \quad (16)$$

$$x_{i,c}^t \text{SoC}_{c,\min} \leq x_{i,c}^t \text{SoC}_c^0 - \sum_{t=1}^{t'} (\Delta T \lambda_i p_{\text{gen},i}^t) \leq x_{i,c}^t \text{SoC}_{c,\max} \quad (17)$$

$$\forall c \in \{E, B\}, i \in \mathcal{N}_E \cup \mathcal{N}_B, t' \in \mathcal{T}$$

Constraints (14) and (15) represent the power limitation of mobile source accession buses. The power is restricted to 0 by $x_{i,c}^t$ until the mobile emergency sources arrive. Once the mobile emergency sources are connected, the power of the accession buses is limited within the rated power range of the mobile generators or energy storage systems, respectively. Constraint (16) shows that the cumulative energy consumption of each mobile generator cannot exceed its remaining energy. Constraint (17) indicates that the cumulative energy consumption of accession bus i for the mobile energy storage system or electric bus should be limited by the SOC limitation of mobile energy storage systems or electric buses.

2) Constraints on Faulted Zones

As stated in Section II, faulted zones are formed when the faults disconnect the zones with the main electrical island. With the faulted components repaired by repair crews, the faulted zones can be energized and the loads can be activated by power from the main electrical island. Assume that in a faulted zone, all lines are connected and the number of buses is equal to that of the lines. In addition, the faulted line is defined as the first line of a zone. For instance, for the faulted zone in Fig. 2, $G_1 = \langle \mathcal{N}_1, \mathcal{E}_1 \rangle$ where $\mathcal{N}_1 = \{30, 31\}$, $\mathcal{E}_1 = \{29 \rightarrow 30, 30 \rightarrow 31\}$, and the first line is $29 \rightarrow 30$.

The operational constraints should be satisfied when a zone is energized after repairing. The constraints are as follows.

$$\sum_{k:k \rightarrow i} \Lambda_{ki}^t + s_i^t = \sum_{j:i \rightarrow j} \Lambda_{ij}^t, \quad i \in \mathcal{N}_l, t \in \mathcal{T} \quad (18)$$

$$\text{sum}(s_i^t + s_{\text{load},i} \gamma_i^t) = s_{s,i}^t, i \in \mathcal{N}_l, t \in \mathcal{T} \quad (19)$$

$$S_{ij}^t = r \text{DIAG}(\Lambda_{ij}^t), i \rightarrow j \in \mathcal{E}_l, t \in \mathcal{T} \quad (20)$$

$$\beta_l^t v_{i,\min} \leq \text{diag}(\mathbf{V}_i^t) \leq \beta_l^t v_{i,\max}, i \in \mathcal{N}_l, t \in \mathcal{T} \quad (21)$$

$$\mathbf{V}_i^t - \mathbf{V}_j^t - (\mathbf{Z}_{ij}^H \mathbf{S}_{ij}^t + \mathbf{S}_{ij}^t \mathbf{Z}_{ij}^H) = 0, \quad i \rightarrow j \in \mathcal{E}_l \setminus \{1\}, t \in \mathcal{T} \quad (22)$$

$$\begin{cases} -(1 - \beta_l^t) M \leq \mathbf{V}_i^t - \mathbf{V}_j^t - (\mathbf{Z}_{ij}^H \mathbf{S}_{ij}^t + \mathbf{S}_{ij}^t \mathbf{Z}_{ij}^H) \\ \mathbf{V}_i^t - \mathbf{V}_j^t - (\mathbf{Z}_{ij}^H \mathbf{S}_{ij}^t + \mathbf{S}_{ij}^t \mathbf{Z}_{ij}^H) \leq (1 - \beta_l^t) M \end{cases} \quad i \rightarrow j = \mathcal{E}_l(1), t \in \mathcal{T} \quad (23)$$

$$\begin{cases} -\beta_l^t (M + \mathbf{j}M) \cdot \text{ones}(3, 1) \leq \Lambda_{ij}^t \\ \Lambda_{ij}^t \leq \beta_l^t (M + \mathbf{j}M) \cdot \text{ones}(3, 1) \end{cases} \quad i \rightarrow j \in \mathcal{E}_l(1), t \in \mathcal{T} \quad (24)$$

$$\frac{x_{l,c}^{n(t-1) - \frac{T_l}{\Delta T_{TS}}}}{N_c} \leq \beta_l^t \leq x_{l,c}^{n(t-1) - \frac{T_l}{\Delta T_{TS}}}, c \in \{R\}, \quad \forall l \in \mathcal{F}, t \in \left\{ \frac{1 + \frac{T_l}{\Delta T_{TS}}}{n}, \dots, \frac{|\mathcal{T}_{TS}| + \frac{T_l}{\Delta T_{TS}}}{n} \right\} \quad (25)$$

$$\beta_l^{t-1} \leq \beta_l^t, \forall l \in \mathcal{F}, t \in \mathcal{T} \setminus \{1\} \quad (26)$$

where $\text{ones}(3, 1)$ represents a 3×1 matrix of ones. Constraints (18)–(20) are the linear approximation of unbalanced power flow constraints [21]. Constraint (18) is the power flow balance constraint of each node in the faulted zone l . s_i^t is the complex power injection at bus i , which depends on the load status and power output at bus i as shown in (19). Constraint (20) represents the relationship between the diagonal elements and non-diagonal elements of S_{ij}^t . Constraint (21) indicates that the bus voltage in faulted zone l is 0 when it is disconnected with the main island and limited when energized. Constraint (22) limits the voltage variation between adjacent nodes for lines except the first line. Constraint (23) shows the voltage variation between adjacent nodes of the first line in zone l by utilizing the Big-M method. $\beta_l^t = 1$ indicates when the first line $i \rightarrow j = \mathcal{E}_l(1)$ is energized, the voltages of node i and j are limited by (23). $\beta_l^t = 0$ indicates line $i \rightarrow j = \mathcal{E}_l(1)$ cannot be energized, then there is no limitation between two bus voltages. Constraint (24) indicates that if the first line is not repaired, the power flow of $i \rightarrow j = \mathcal{E}_l(1)$ would

be 0. Constraint (25) shows the relationship between the arrival of the repair crews and the availability of the faulted component, where T_l is the time needed for repairing the faulted component. Constraint (26) indicates that once the faulted component is repaired, zone l is energized and the status will not change anymore.

Other operational constraints in the PDS, including unbalanced three-phase power flow equations, voltage limits, and power output and energy limits of local DGs, as well as radial topological constraints, can be accessed in the authors' previous study [8].

To sum up, the load restoration problem considering the PDS and TS interdependency is formulated as a MILP model, which can be solved readily using off-the-shelf optimization solvers.

E. Discussion

When building the dynamic load restoration model, some measurement data are needed, such as the load demand before the outage, the availability of lines and sources, etc. These data are collected by measurement devices and inevitably contain some bad data, which would affect the feasibility of the obtained strategy. The collected measurement data can be filtered to obtain the authentic information using robust estimation methods, such as those in [22] or [23], to ensure the effectiveness of the proposed model.

IV. NUMERICAL RESULT

In this section, we validate the effectiveness of the proposed model in a modified 32-node PDS coupled with a modified 12-node TS [8], as shown in Fig. 2. The MILP is modeled using a CVX toolbox [24] on a MATLAB 2018b and the solver MOSEK [25] is used to solve the model. All simulations are carried out on a personal computer with Intel Core I7 CPU at 3.6 GHz and 8 GB of RAM.

A. Test System Information

The schematic diagram of the TS with critical roads is shown in Fig. 2. The system is circular which is similar to the traffic network of Beijing. The parameters of the TS are shown in Table I. The free speed indicates the maximum speed allowed in the road.

TABLE I
PARAMETERS OF THE TS

Links	Length (km)	Free/Max. speed (km/h)	Max. flow (vph)
1-2, 6-10, 12-11, 7-3	6	72	1440
1-3, 2-6, 10-12, 11-7	3	72	1440
4-5, 5-9, 9-8, 8-4	6	72	720
1-4, 3-4, 2-5, 5-6, 9-10, 9-12, 8-11, 8-7	3	72	720

In the TS, the origin and destination pairs (O-D pairs) of different types of MERs are listed in Table II.

In Table II, the entry “number” indicates the number of vehicles of the corresponding type of MERs. The emergency center can dispatch 2 mobile generators, 2 mobile energy storage systems, 1 repair crew, and 6 electric buses. In addition,

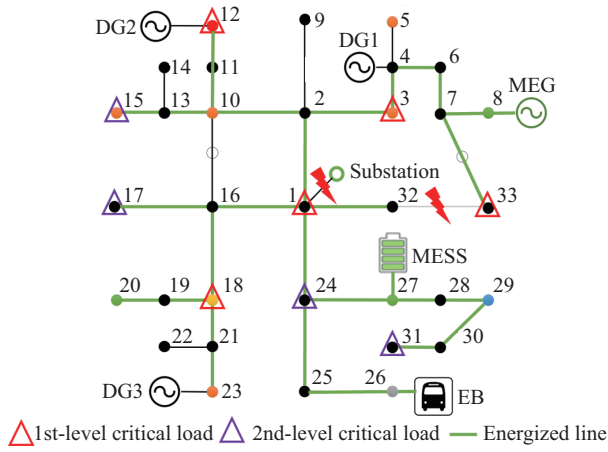


Fig. 8. The post-restoration state of the PDS.

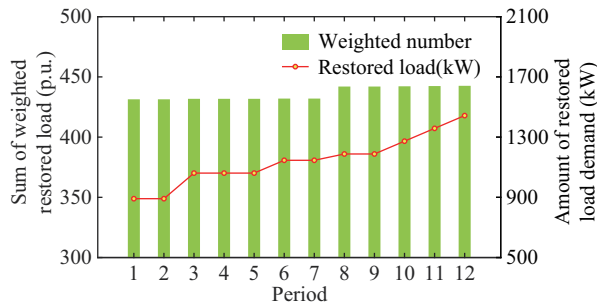


Fig. 9. The restoration status of loads in all periods in the PDS.

During the first 7 periods, all 1st and 2nd level loads except the 2nd level load at 31 are restored. In period 8, the repair crews repair the faulted component so the faulted zone can be energized after that. The outputs and energy consumptions of all sources are illustrated in Figs. 10 and 11 respectively.

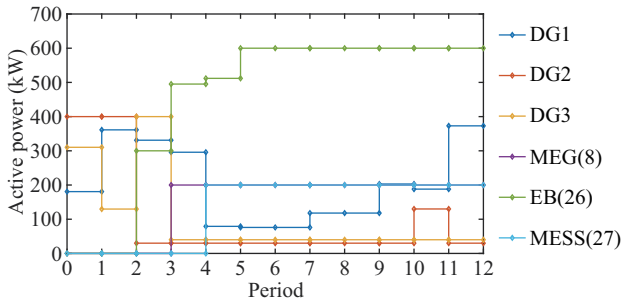


Fig. 10. Power outputs of power sources.

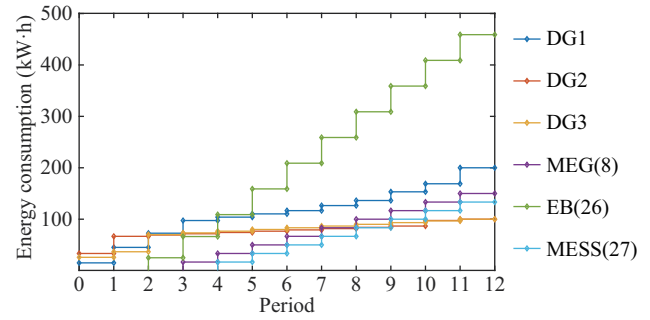


Fig. 11. Energy consumptions of power sources.

According to Figs. 10 and 11, at the early stages, the loads are restored by local DGs. With the accession of mobile power sources, they primarily serve the loads, and the energy of the local DGs gradually runs out.

3) Comparison with Method without Considering Interdependency

In this part, the results of the method without considering interdependency are obtained. In the method without considering interdependency, the restoration status of traffic lights is first obtained by solving the service restoration problem in the PDS supported by local DGs, following which the dispatch strategy of MERs is determined, then the final restoration strategy is obtained. The specific models can be accessed in [8].

After 14.97 s, the restoration results are obtained. The restoration status of some traffic lights is shown in Fig. 12, the dispatch strategy of MERs in Table V, and the load restoration comparison results of the two methods in Fig. 13.

TABLE V
THE ROUTING AND ARRIVAL TIME OF VEHICLES IN THE COMPARISON CASE

Type	O-D pair	Routing	Travel time (min)	Number
mobile emergency generator	1-6	1-2-6	10	2
mobile emergency generator	1-7	/	/	0
mobile energy storage system	1-9	1-4-8-9	18	2
repair crew	1-10	1-2-6-10	15.5	1
electric bus	13-12	13-5-9-12	10	1
electric bus	14-12	14-3-4-8-9-12	23.5	2
electric bus	15-12	15-12	6	3

Using the method without considering PDS and TS interdependency, the traffic lights at node 4, 7, 8, 9, and 10 would not be restored at the beginning due to the limited generation resources of the local DGs, which further influence

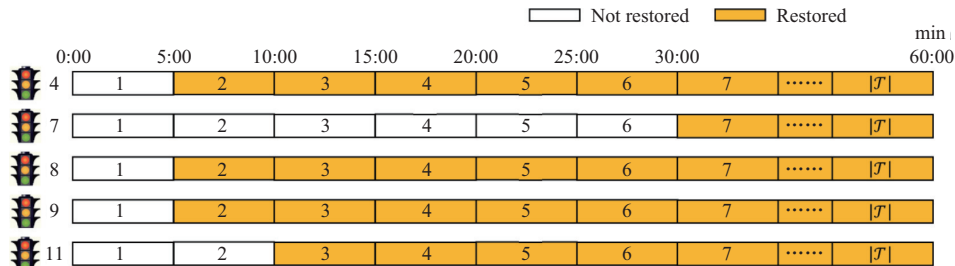


Fig. 12. The restoration status of some traffic lights.

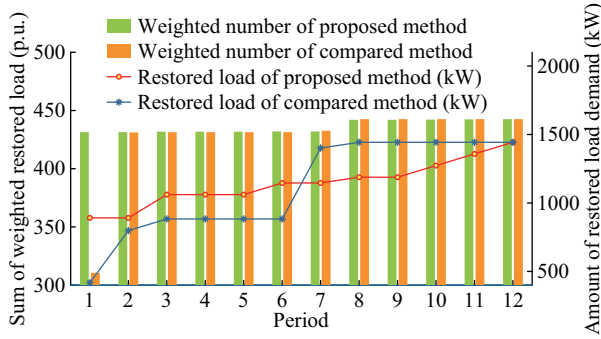


Fig. 13. The restoration status of loads in all periods of two methods.

the MERs dispatch in the TS. The mobile energy storage systems and electric buses from 13 to 12 are delayed due to the outage of traffic lights at 7 and 4. Furthermore, the load restoration is affected by the delay of the MERs. As shown in Fig. 13, using the method without considering interdependency, some critical loads are not restored in period 1 as the energy remaining in the local DGs cannot supply all critical loads for a very long time. At the early stage, i.e., the periods 1 to 6, the amount of restored loads is lower than the proposed method. From period 7, the amount of restored loads is higher because the generation capacity is increased as the mobile emergency sources are accessed by the PDS. In addition, the energy is relatively sufficient as energy consumption is lower at the early stage. At period 12, the load restoration status of two methods are the same. In the strategy without considering interdependency, the main objective value is 5115.2 and the restored energy is 1113.67 kW·h. Using the proposed method considering interdependency, the total restored energy increases 2.57%.

V. CONCLUSION

This paper proposes a dynamic load restoration method considering the PDS and TS coupled by traffic lights and MERs. By solving the proposed MILP, the dispatch strategy of MERs and the dynamic load restoration strategy are obtained with the efficiency of both the TS and PDS maximized. Test results show that without considering interdependency, some traffic lights are not restored in time, then the MERs will delay and further influence load restoration in the PDS. The proposed approach can obtain better dispatch and restoration strategies, validating the effectiveness of the proposed method.

APPENDIX

The corresponding nodes between the PDS and TS in Fig. 2 are listed in Table AI.

TABLE AI
THE CORRESPONDING NODES BETWEEN PDS AND TS

Node ID in TS	Node ID in PDS	Node ID in TS	Node ID in PDS
1	12	7	20
2	5	8	18
3	15	9	27
4	10	10	29
5	3	11	23
6	8	12	26

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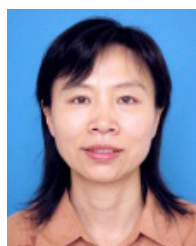
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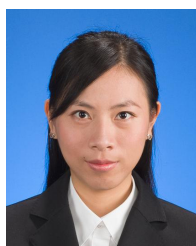
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